Lecture 26. Radiative and radiative-convective equilibrium.

Objectives:

1. Radiative equilibrium models.

2. Radiative-convective equilibrium models.

Appendix. Derivation of the Eddington gray radiative equilibrium.

Required reading:

L02: 8.3

1. Radiative equilibrium models.

These models predict the atmosphere temperature profile of an atmosphere in radiative equilibrium $\frac{dF_{net}}{dz} = 0$

Under the "gray atmosphere" assumption, we can solve for the temperature profile analytically

Eddington gray radiative equilibrium results: (see Appendix for the full derivation).

Assumptions:

1) Radiative equilibrium: $\frac{dF_{net}}{dz} = 0$
2) Gray atmosphere in longwave
3) No scattering and black surface in longwave
4) No solar absorption in the atmosphere
5) Eddington approximation: $I(\mu) = I_0 + I_1\mu$

Eddington approximation $\rightarrow \quad F_{net} = \frac{4\pi}{3} I_1$

Integrate RTE over $d\mu$: $2\pi \int_{-1}^{1} \left[\mu \frac{dI}{d\tau} = I - B\right] d\mu$

$$\frac{dF_{net}}{d\tau} = 4\pi I_0 - 4\pi B \quad \rightarrow \quad I_0 = B$$

Integrate RTE over $\mu d\mu$: $\frac{4\pi}{3} \frac{dI_0}{d\tau} = F_{net}$

$$B(\tau) = B(0) + \frac{3}{4\pi} F_{net}\, \tau$$

Constants $B(0)$ and F_{net} determined from boundary conditions.

Top: 1) zero incident longwave flux,

2) outgoing longwave flux equals absorbed solar flux $F_{sun} = \frac{S}{4}(1 - \bar{r})$.

Bottom: downwelling SW + LW flux equals surface emission.

Longwave flux profile:

$$F^{\uparrow}(\tau) = F_{sun}(1+\frac{3}{4}\tau) \quad \text{and} \quad F^{\downarrow}(\tau) = F_{sun}(\frac{3}{4}\tau) \qquad [26.1]$$

where $F_{sun} = (1-\overline{r})F_0 / 4$

Atmosphere blackbody emission and temperature profiles:

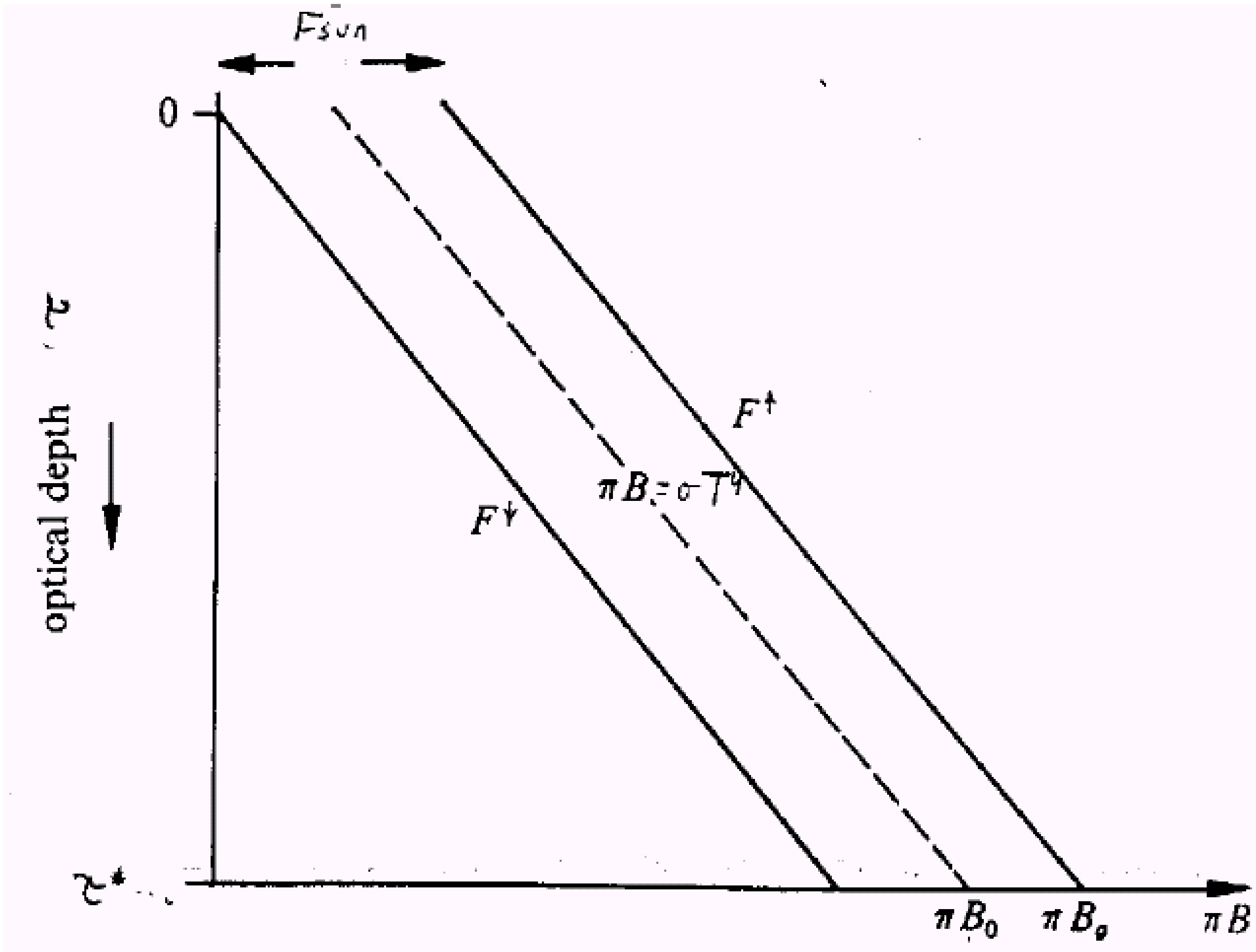
$$B(\tau) = \frac{F_{sun}}{2\pi}(1+\frac{3}{2}\tau) \quad \text{and} \quad T^4(\tau) = T_e^4(\frac{1}{2}+\frac{3}{4}\tau) \qquad [26.2]$$

Surface temperature is discontinuous with the atmosphere (hotter):

$$B_s = B(\tau^*) + \frac{F_{sun}}{2\pi} \quad \text{and} \quad T_s^4 = T_e^4(1+\frac{3}{4}\tau^*) \qquad [26.3]$$

Implications:

- ✓ Greenhouse effect – larger $\boldsymbol{\tau}^*$ increases surface temperature
- ✓ Runaway greenhouse effect - $\boldsymbol{\tau}^*$ increases => T_s increases
- ✓ Positive feedback – higher temperature => greenhouse gases



Profiles of upwelling, downwelling, and emitted flux for gray radiative equilibrium. [Houghton, 1986]

Eddington gray radiative equilibrium temperatures

If one wants the temperature profile in terms of height, one needs to relate optical depth to height.

Assume that an absorber has the exponential profile

$$\rho_a = \rho_0 \exp(-z / H_a) \qquad [26.4]$$

So the profile of optical depth is

$$\tau(z) = \overline{k}_a \int_z^{\infty} \rho_a(z)dz = \overline{k}_a \rho_0 H_a \exp(-z / H_a) = \tau^* \exp(-z / H_a) \qquad [26.5]$$

Temperature profile

$$T^4(z) = T_e^4 (1 + \frac{3}{4}\tau^* \exp(-z / H_a)) \qquad [26.6]$$

Lapse rate

$$\frac{dT}{dz}(z) = -\frac{3}{8}\frac{\tau^*}{1+\frac{3}{2}\tau^*}\frac{T(z)}{H_a}\exp(-z / H_a) \qquad [26.7]$$

Implications:

- ✓ Low **τ*** => stable atmosphere
- ✓ Smaller scale height Ha of the absorber causes steeper lapse rate
- ✓ Steepest lapse rate near the surface (z=0)

2. Radiative-convective equilibrium models.

- These climate models solve for the vertical profile if temperature using accurate broadband radiative transfer models.
- Model inputs vertical profile of gases, aerosols and clouds. Iterates the temperature profile to archive equilibrium (i.e., zero heating rates or $\frac{dF_{net}}{dz} = 0$)
- Climate feedbacks can be included by having water vapor, surface albedo, clouds, etc. depend on temperature.

Solving for radiative equilibrium:

Iterate the temperature profile T(z) to get zero heating rates $\frac{\partial T}{\partial t} = 0$

1. ***Time marching method***:

   T at t+1 time step from heating rate at time t:

$$T^{t+1}(z_k) = T(z_k) + \left(\frac{\partial T(z_k)}{\partial t}\right)^t \Delta t \qquad [26.8]$$

2. ***Direct solve***:

   Use gradient information in nonlinear root solver (faster, but more complex than time marching)

Radiative equilibrium temperature profiles show (see Figure 26.1 below):

- ✓ CO2 –only-atmosphere has less steep profile.
- ✓ Earth's stratosphere warms due to UV absorption by ozone.
- ✓ Most greenhouse effect from water vapor.

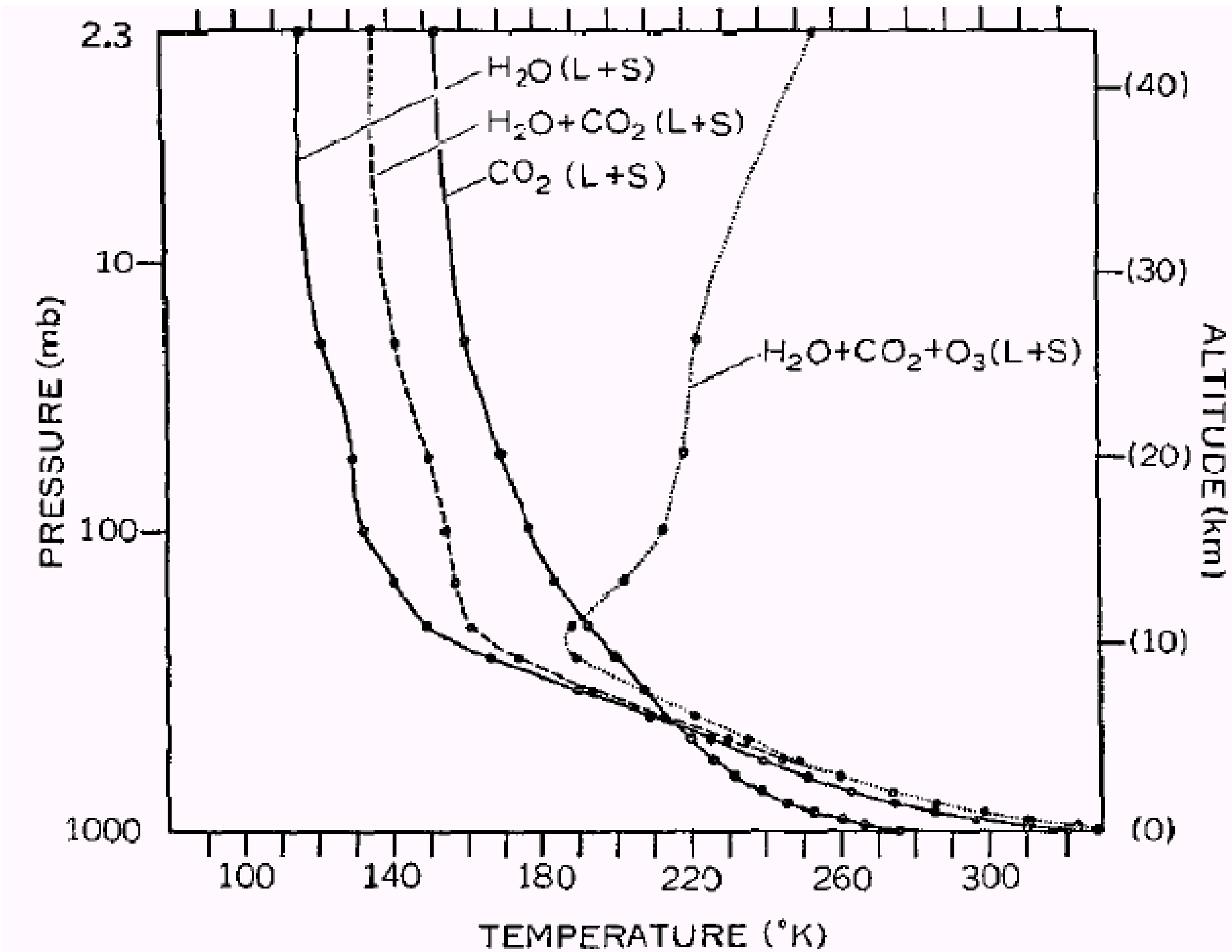


Figure 26.1 Pure radiative equilibrium temperature profiles for various atmospheric gases in a clear sky at 35 N in April. L+S means that the effects of both longwave and shortwave radiation are included (from Manabe and Strickler, 1964).

Results: the radiative equilibrium surface temperature is too high and the temperature profile is unrealistic.

Problem: radiative equilibrium surface temperature lapse rate near the surface exceeds threshold for convection

Fix: assume convection limits lapse rates to < γ_c (e.g., 6.5 K/km)

- ✓ **Radiative-convective equilibrium is equilibrium of radiative+ convective fluxes**

Convective adjustment methods:

1. Move heat like convection: if γ_c exceeded, adjust temperature so γ_c achieved and heat is conserved
2. Parameterize convective flux, e.g.

$$F_{conv} = C\left(\left|\frac{dT}{dz}\right| - \gamma_c\right) \quad \text{if} \left(\left|\frac{dT}{dz}\right| - \gamma_c\right) > 0 \qquad [26.9]$$

Results of the RCE model developed by Manabe and Strickler (1964):

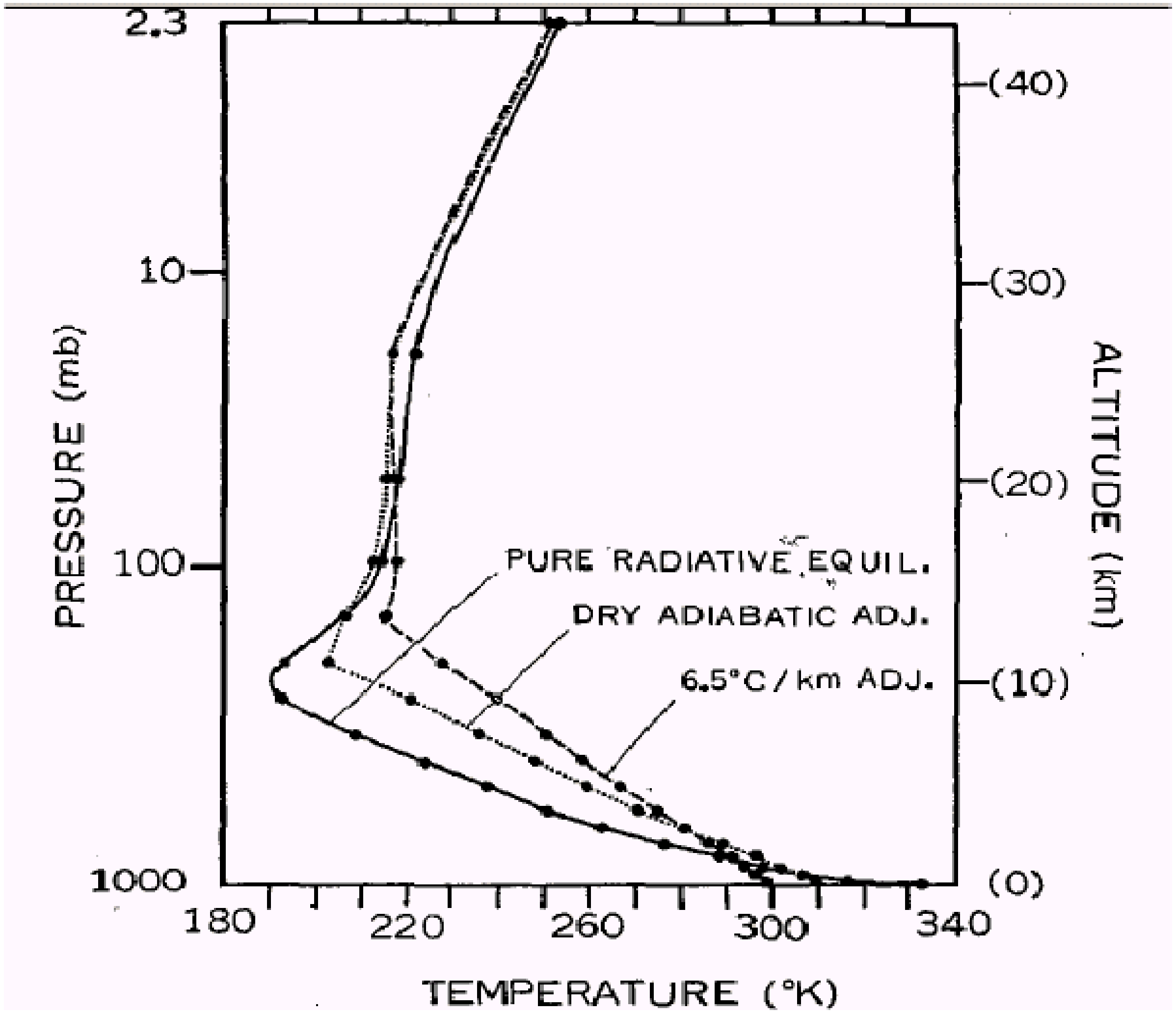


Figure 26.2 Pure radiative equilibrium and radiative-convective equilibrium temperature profiles for two values of γ_c for clear sky.

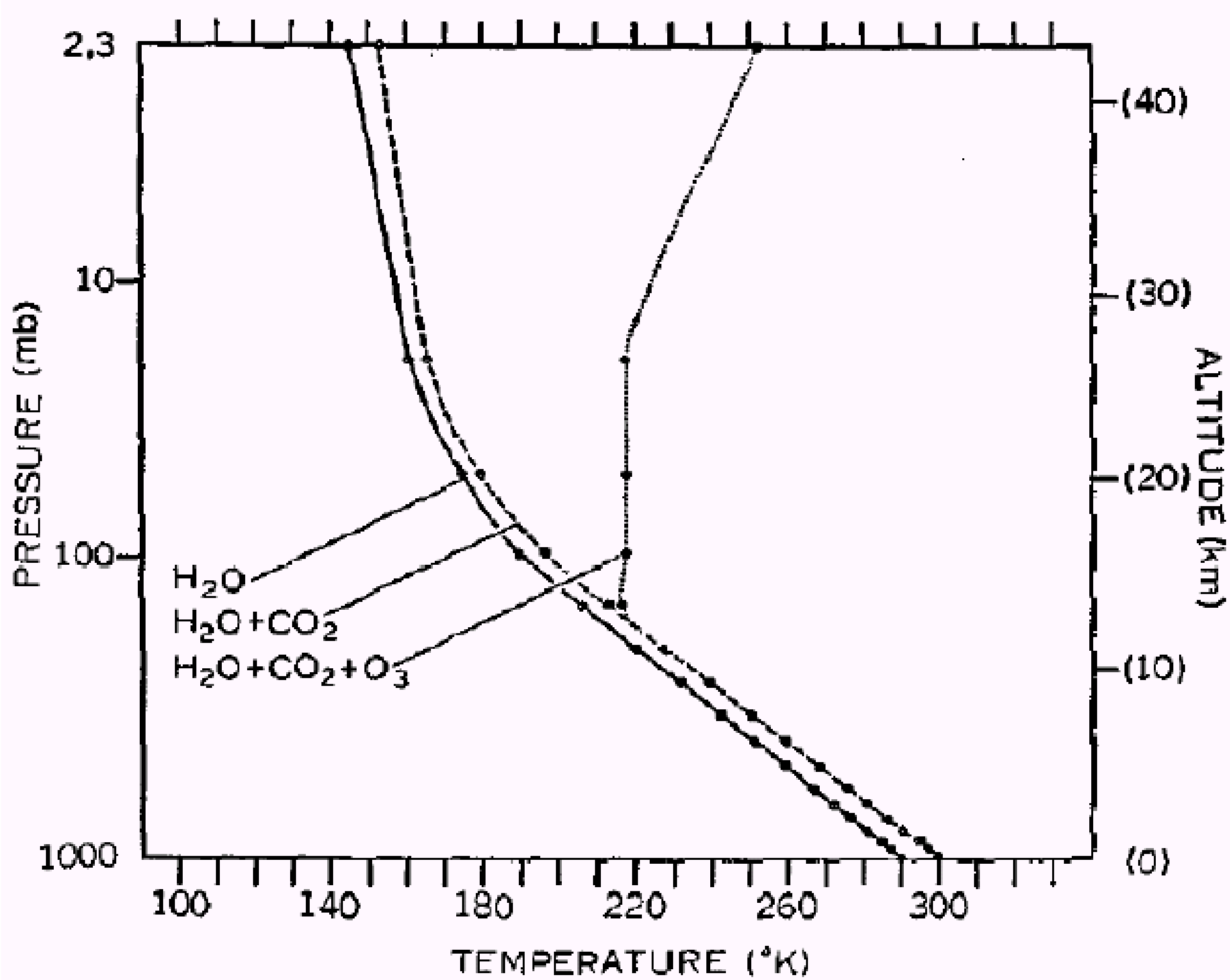


Figure 26.3 Radiative-convective equilibrium temperature profiles for various atmospheric gases in a clear sky at 35 N in April.

Comparing figures 26.1 and 26.3:

- ✓ Radiative equilibrium is fairly accurate for the stratosphere (though latitudinal and seasonal dependence is not correct)
- ✓ Convection required for reasonable tropospheric temperatures

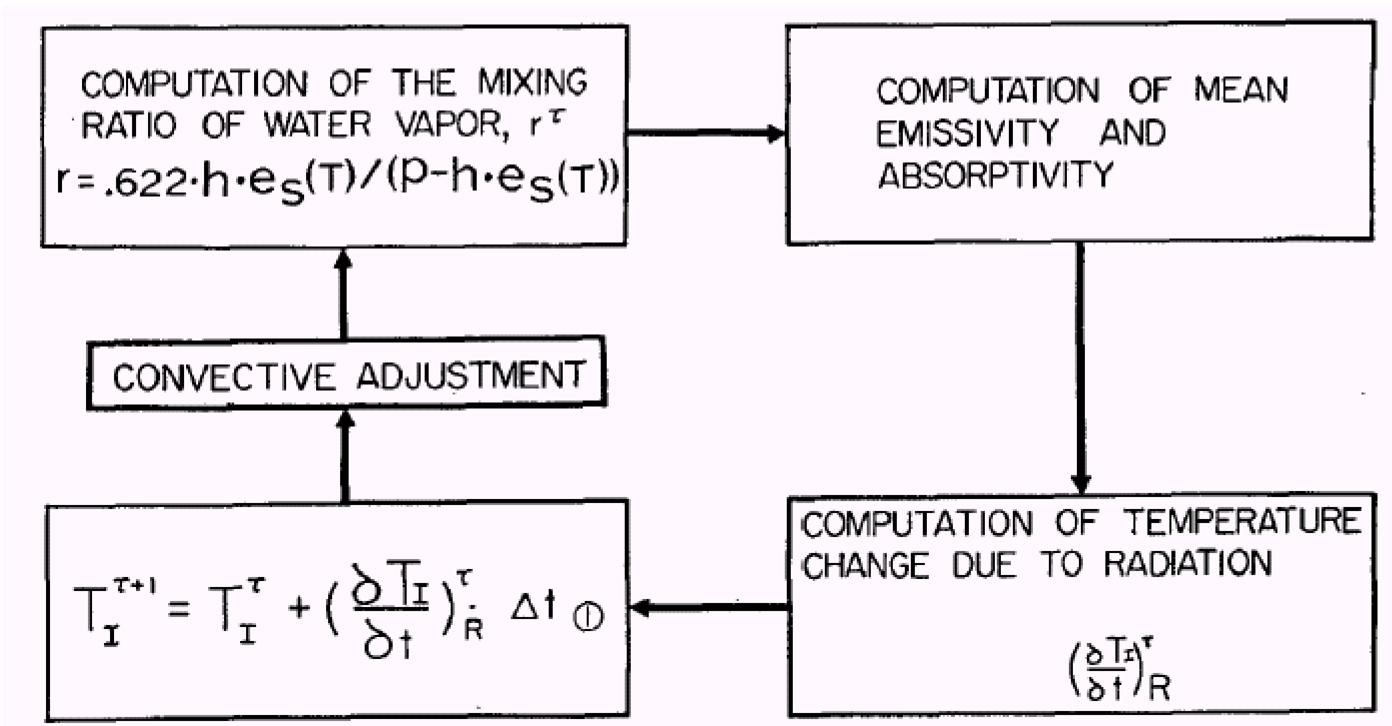
Water Vapor Feedback with RCE Model

Manabe and Wetherald (1967) assumed fixed profile of relative humidity (linearly decreasing with height in troposphere). Actual water vapor feedback is more complicated (dynamics, clouds, precipitation, etc.).

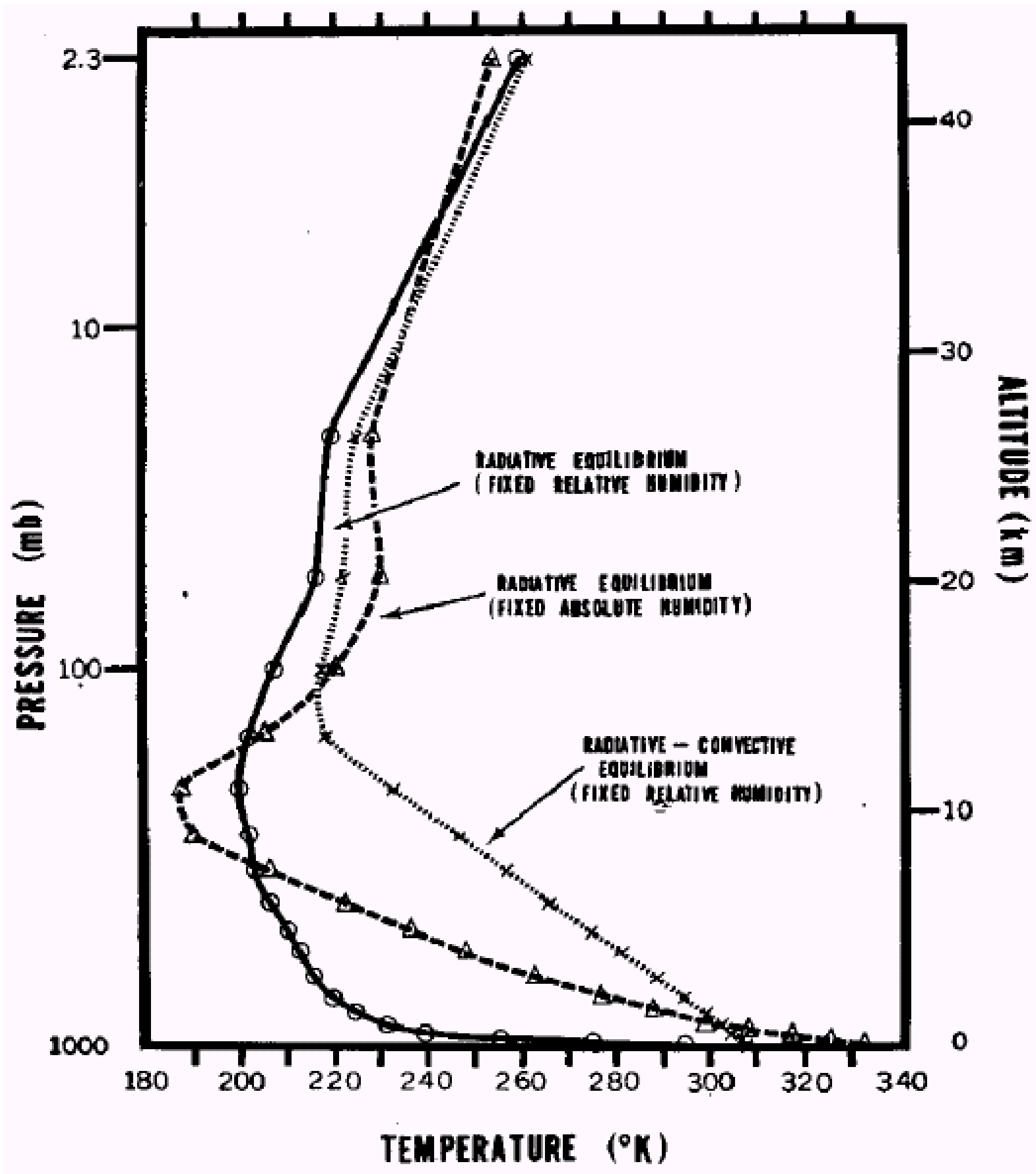
Warming → more water vapor → higher atmos emissivity
→ more greenhouse effect → more temperature increase.

Gives much steeper radiative equilibrium lapse rate than fixed absolute humidity.

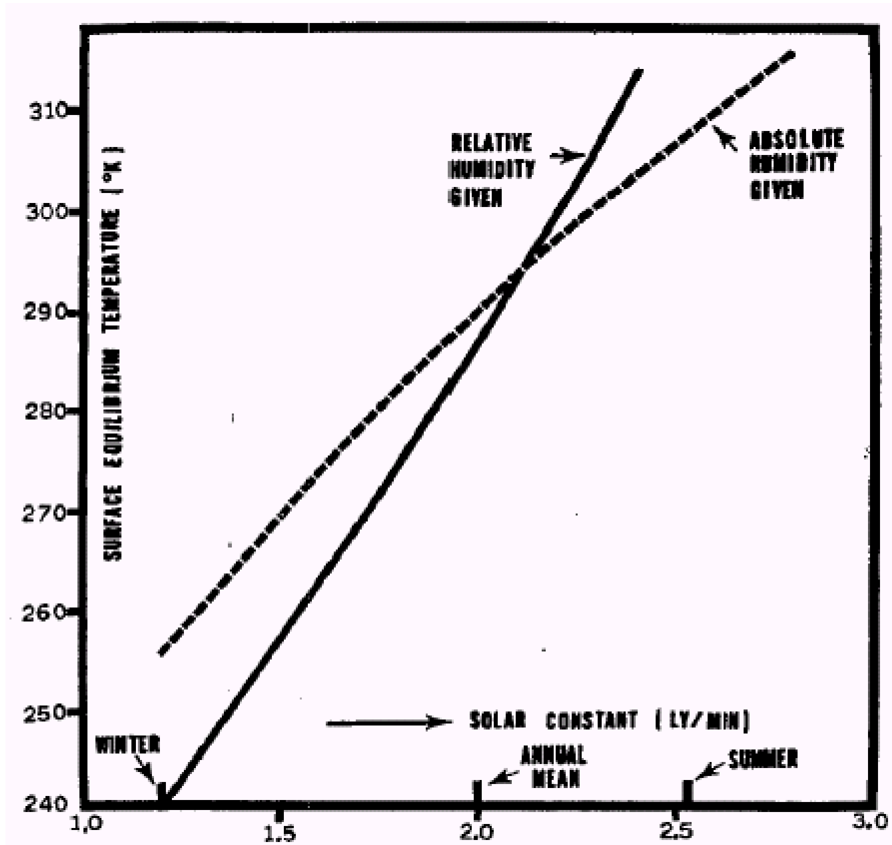
RCE shows increased climate sensitivity from water vapor feedback:
larger surface temperature change for same radiative forcing.



Flowchart for the time-marching numerical integration. [Manabe and Wetherald, 1967]



Temperature profile for pure radiative equilibrium with fixed absolute humidity, pure radiative equilibrium with fixed relative humidity and radiative-convective equilibrium with fixed relative humidity (all clear sky) [Manabe and Weatherald, 1967]



Climate sensitivity in terms of surface temperature versus solar constant for fixed absolute humidity (no climate feedbacks) and for fixed relative humidity (water vapor feedback). [Manabe and Wetherald, 1967]

TABLE 4. Equilibrium temperature of the earth's surface (°K) and the CO_2 content of the atmosphere.

CO_2 content (ppm)	Average cloudiness: Fixed absolute humidity	Average cloudiness: Fixed relative humidity	Clear: Fixed absolute humidity	Clear: Fixed relative humidity
150	289.80	286.11	298.75	304.40
300	291.05	288.39	300.05	307.20
600	292.38	290.75	301.41	310.12

TABLE 5. Change of equilibrium temperature of the earth's surface corresponding to various changes of CO_2 content of the atmosphere.

Change of CO_2 content (ppm)	Fixed absolute humidity: Average cloudiness	Fixed absolute humidity: Clear	Fixed relative humidity: Average cloudiness	Fixed relative humidity: Clear
300 → 150	−1.25	−1.30	−2.28	−2.80
300 → 600	+1.33	+1.36	+2.36	2.92

***Appendix*. Derivation of the Eddington gray radiative equilibrium.**

Assumptions:

6) Radiative equilibrium: $\dfrac{dF_{net}}{dz} = 0$

7) Gray atmosphere in longwave

8) No scattering and black surface in longwave

9) No solar absorption in the atmosphere

10) Eddington approximation: $I(\mu) = I_0 + I_1\mu$

Since the atmosphere is gray (all wavelength are equivalent), one can write the wavelength integrated thermal emission radiative transfer equation (no scattering)

$$\mu \frac{dI}{d\tau} = I - B$$

where $\boldsymbol{I}$ is the integrated radiance (W m^{-2} st^{-1}), $\boldsymbol{\tau}$ increases downward , and $\boldsymbol{\mu} > 0$ in the upward direction. Note that deriving the variation of B with the optical depth $\boldsymbol{\tau}$ is equivalent to determining the temperature profiles since the blackbody emission is a function of temperature only.

Using the Eddington approximation, the net flux (positive upward) becomes

$$F_{net} = 2\pi \int_{-1}^{1} I\mu d\mu = \frac{4\pi}{3} I_1$$

The radiative equilibrium assumption implies that F_{net} (and I_1) is constant with optical depth.

Integrating the above radiative transfer equation over dμ gives

$$2\pi \frac{d}{d\tau} \int_{-1}^{1} I\mu d\mu = 2\pi \int_{-1}^{1} I d\mu - 2\pi \int_{-1}^{1} B d\mu$$

$$\frac{dF_{net}}{d\tau} = 4\pi I_0 - 4\pi B$$

Under the radiative equilibrium assumption, we have

$$I_0 = B$$

Integrating the radiative transfer equation over μdμ gives

$$2\pi \frac{d}{d\tau} \int_{-1}^{1} I\mu^2 d\mu = 2\pi \int_{-1}^{1} I\mu d\mu - 2\pi \int_{-1}^{1} B\mu d\mu$$

Since B is isotropic the last term drops out leaving

$$\frac{4\pi}{3}\frac{dI_0}{d\tau} = F_{net} = \frac{4\pi}{3} I_1$$

$$\frac{dB}{d\tau} = I_1$$

Thus, the solution for B is simply a linear function of optical depth:

$$B(\tau) = B(0) + I_1\tau$$

Constants B(0) and I_1 need to be determined from the boundary conditions.

Top of the atmosphere:

First boundary condition: no thermal downwelling flux

$$F^{\downarrow}(0) = 2\pi \int_{-1}^{0} I\mu d\mu = \pi B(0) - \frac{2\pi}{3} I_1 = 0$$

so we have

$$I_1 = \frac{3}{2} B(0) \quad \text{or} \quad F_{net} = 2\pi B(0)$$

Second boundary condition: upwelling longwave flux is equal to the absorbed solar flux F_{sun}:

$$F^{\uparrow}(0) = 2\pi \int_0^1 I\mu d\mu = \pi B(0) + \frac{2\pi}{3} I_1 = F_{sun}$$

(Recall that the absorbed solar flux F_{sun} is $F_{sun} = (1-\overline{r})F_0 / 4)$)

Putting in $I_1 = \frac{3}{2} B(0)$ gives

$$F_{sun} = 2\pi B(0) = F_{net}$$

So now we have the B(0) and I_1 and thus the atmosphere Planck function profile is determined

$$B(\tau) = \frac{F_{sun}}{2\pi}(1 + \frac{3}{2}\tau)$$

The final step is to apply the boundary condition at the surface to obtain the surface temperature T_s. This boundary condition is that the emitted flux by the surface equals to the sum of the downwelling shortwave and longwave flux at the black surface:

$$F_{sun} + F^{\downarrow}(\tau^*) = \pi B_s$$

where $F^{\downarrow}(\tau^*) = \pi B(\tau^*) - \frac{2\pi}{3} I_1$

Using $F_{sun} = \frac{4\pi}{3} I_1$ gives the emission from the surface

$$B_s = B(\tau^*) + \frac{F_{sun}}{2\pi}$$

which is discontinuous with the atmospheric emission.

The previous results can be expressed in terms of temperature by

$$T^4(\tau) = T_e^4(\frac{1}{2} + \frac{3}{4}\tau)$$

where $\sigma T_e^4 = F_{sun}$

$$T_{top}^4 = \frac{1}{2} T_e^4$$

$$T_s^4 = T_e^4(1 + \frac{3}{4}\tau^*)$$

For $F_0 = 1366$ W/m^2 and $\overline{r} = 0.3$:

$T_e = 255$K and a "top" temperature $T_t = 214$ K

Assuming a global averaged surface air temperature of $T(\tau^*) = 288$ K gives a gray body optical depth of $\tau^* = 1.5$, and a surface skin temperature of $T_s = 308$ K